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Numerical Scheme to Simulate Caputo Fractional Derivative-Based Unemployment Model

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Short Communication

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Abstract

The study constructs a novel numerical scheme through the generalized Adams-Bashforth-Moulton Predictor Corrector method for the simulation of Caputo fractional derivative unemployment model. In developing the numerical scheme, the developed system is primarily summarised in the procedure of a fractional order of differential equations, and further described in the form of Volterra integral to help achieve the iterative formula. The application of the numerical scheme to the Caputo derivatives-based unemployment model yields the system's discretized form for simulation purposes.

Keywords: Numerical scheme; caputo fractional derivative; adams-bashforth-moulton; predictor corrector; volterra integral; iterative.

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1 Introduction

There have been many models developed in the area of unemployment, with (Misra and Singh 2011) and (Misra and Singh 2013) developing and analysing a nonlinear mathematical model to control unemployment based on some concept adopted from a developed model which considered a housing allocation of homeless families as a result of natural disaster by (Nikolopoulos and Tzanetis 2003). They made the assumption that, every migrant to the unemployment chamber is qualified and capable of doing any assigned work. They further assumed that, there is a constant rate of increase in the number of unemployment. During the modelling process, they compartmentalized their model into number of individuals who are employed, unemployed individuals and the new job openings or vacancies created at any time.

The work of (Misra and Singh 2011, Misra and Singh 2013) served as a motivation for many researchers (Pathan and Bathawala 2015). were motivated by the work done by (Misra and Singh 2011). This led them to propose a model to better understand unemployment problem and how it could easily be solved with self-employment as an intervention. However, their model failed to consider any time delay by the private sector and the government in creating additional vacancies. Additionally, (Munoli and Gani 2016) were also motivation by the work of (Misra and Singh 2011, Munoli and Gani 2016) developed a mathematical nonlinear unemployment model and analysed the optimality of the cost of new vacancies. They considered death and retirement of employed individuals as vacancies, and also introduced two control variables in their optimal control problem. These two optimal control variables introduced in the model were, government’s implemented policies to create employment from the unemployed individuals, and government creating vacancies.

Moreover, in addressing the issue in poor countries with scanty resources, (Al-Maalwi et al. 2021) developed a mathematical model comprising of three variables. The variables were number of people that are employed, unemployed persons and available vacancies. They discovered that the government may effectively address the issue of unemployment by increasing work possibilities and lowering the rate of decline. They determined that, in doing so, it will raise the employment rate, lower the rise in unemployment, and ultimately raise the unemployment reduction rate.

According to (Ashi et al. 2022) addressing the issue of unemployment depends solely on the support from the public and private sectors. The work noted that, this support must be expressed through raising the employment rate in accordance with the creation of new job opportunities for both new entrants and the unemployed in the labor market.

Moreover, (Mallick and Biswas 2017) also developed and analysed an unemployment non-linear model. This model was embedded within it with two policies. Thus, providing skills manpower to the populace and creating of new vacancies. They analysed their developed model with different constant control strategies with a comparison to the results obtained with optimal control variable after the usage of Pontryagin’s Maximum Principle. They unravelled that, a government’s policy that creates more vacancies is the ideal situation than the government not embarking on any policy. They further observed that government’s policy to provide skilled manpower is very effective to maximising the objective function than just the policy of creating new vacancies as a result of death and retirement as adopted in the work of (Munoli et al. 2017, Mallick and Biswas 2017) additionally controlled both policies and identified that, when both policies are well controlled by the government, there is a greater minimised value of the objective function than when the government does not control the policies. Additionally, they deduced that there is a minimised value in the number of unemployed persons when both policies were controlled with the maximised value of the objective function. In the end, they came to the conclusion that government should set and oversee the policies in order to reduce the number of jobless individuals.

In a similar work done by (Ayoade et al. 2020) on the impact of vocational training on unemployment, their work developed a four compartmental unemployment model with four systems of first order ordinary differential equations for each of the compartments. The compartments were the theoretical compartment, which represents persons not participating in vocational education program as a result of hostile policy and/or its implementation by government, but engaging in theoretical form of education, the second compartment was vocational compartment being persons into vocational education with the aim of creating vacancies for themselves after the training, the third was unemployed compartment which was also those unemployed persons as a result of the lack of vocational training or lack of startup capital despite their vocational skills or training, and finally the employed compartment which represented the employed persons as a result of self-employment after the vocational training or were being employed by others. Their model assumed that, the rate of disengaging from theoretical and vocational education, and the rate of migration to foreign countries for employment opportunities is the same, and that, the employed persons do not lose their jobs except they resign from their jobs in search of better opportunities. It is worth noting that, the assumption of the model never factored the scenario when there is low skill acquisition of the vocational trainees resulting in their inability to be self-employed, and also the bidirectional effect of those losing interest in theoretical education and going back to vocational education. Additionally, (Ayoade et al. 2020) did not take fractional order derivatives into account while determining the threshold for the implementation of vocational education to be successful using epidemic models. They also concluded that the underutilization of vocational training contributes to Nigeria's high unemployment rate.

(Nti et al. 2024) developed a fractional unemployment model to understand the dynamics of unemployment in Ghana. The choice of the fractional derivative to that of the classical or traditional models was as a result of the after-effect or hereditary characteristics and nuance nature of fractional calculus models with (Omame et al. 2022) adding that, in many situations, nonlinear and integer order models are not really acceptable in many frameworks. Again, (Rosenfeld and Dixon 2017 and Toufik and Atangana 2017) posited that, many types of fractional derivatives have been used in many fields. However, the fractional derivative that is mostly used is the Caputo fractional derivative due to its requisition of initial conditions in the form of integer order, which can mostly be associated to physical quantities. Based on the advantages, (Nti et al. 2024) in developing a Caputo unemployment model to understand the dynamics of unemployment, considered four compartments. Thus. The unemployment compartment (U), employed compartment (E), skill training compartment (A), and new vacancies compartment (V). The work classified graduates from skill training into two categories (high skilled and low skilled) and added that, some of the high skilled trainees are supported with startup capital to create vacancies for themselves while the low skilled trainees fall back into the unemployment compartment. The model further considered the impact of government revenue on job creation through provision of logistics for the training institutions, and tax cuts or holidays for the employed to enable them invest into the economy to yield a positive ripple effect of the employed creating more vacancies for others, while institutions that enjoy tax holidays or tax cuts can invest into expansionary strategies of the company to help create vacancies in the existing industries.

2 Literature Review

Researchers have devoted considerable time and decades to study fractional calculus with fractional differential equations (PDEs) proving to be essential and superior models for numerous physical systems due to the after effect or memory of these physical phenomena or dependence of all past periods instead of just an instant (Momenzadeh and Ahrabi 2017). (Momenzadeh and Ahrabi 2017) further noted that, regular usage of PDEs have been enormous in the mathematical research community, notwithstanding, nonlinear fractional differential equations might have an impossible analytical solution. This impossibility compels researchers to develop numerical schemes from traditional approaches through an appropriate modification to be able to handle the fractional derivative operators.

Contrasting the integer order derivatives to the Caputo derivative (Diethelm et al. 2004) and (Malinowska and Torres 2012) noted that, there is a problem when approximating the Caputo derivative of a function as a result of the nonlocal property of the fractional order derivative. Hence, the Caputo fractional order derivative cannot be computed with only the local information about the point. In this case, the integral in the Caputo fractional derivative demands the usage of the whole history of a function in computing the fractional derivative (Rosenfeld and Dixon 2017).

There have been many numerical methods like Adams-Bashforth-Moulton (ABM) predictor corrector, fractional differential transforms, forward Euler, difference method, fractional linear multistep method, fractional backward differentiation formulae among others applied in solving both linear and nonlinear fractional order. However, (Momenzadeh and Ahrabi 2017) found out that, ABM predictor corrector method outperforms the generalized differential transform method and its enhancement being the multi-step generalized differential transform method. Also, (Roman 2021) presented a numerical scheme of the Cauchy problem through a model class of fractional linear oscillators and found out that, the Adams-Bashforth-Moulton (ABM) was more accurate and converges faster than the explicit nonlocal finite-difference scheme.

(Aminikhah et al. 2018) posited that, fractional Adams-Bashforth-Moulton is a strong method for solving ordinary differential equations and has the ability to handle numerous chaotic models with (Shabani et al. 2021). adding that, this method is very stable for models of chaotic nature and other fractional ordinary differential equations models. Also, among the numerous numerical treatments of fractional differential equations, the work of (Ford and Connolly 2006) underscored that, the ABM predictor corrector method by (Diethelm and Ford 2002) is mostly recommended due to the simplicity in its implementation in both linear and nonlinear fractional differential equations while (Rosa and Torres 2023) also noted that, the predictor corrector method compared to the Euler's method produces better approximations.

Moreover, the work of (Ford and Connolly 2006) reviewed and compared five numerical methods that are used to solve single term fractional differential equations using the criteria of consistency of the method, convergence of the method and the stability of the method. They found that, out of the five methods being the method of Implicit Quadrature which was introduced by (Diethelm 1997) the method of collocation also described by (Blank 1997) the method of Approximate Mittag-Leffler which was also considered in the work of (Diethelm and Luchko 2004) the method of Finite Difference found in the work of (Gorenflo 1997) and finally the predictor corrector method by (Diethelm et al. 2002) the predictor corrector is better in most cases on the basis of convergence, consistency and stability with other advantages of the easiness of its implementation for nonlinear systems of equations as well as the ability to be nearly optimal at the point when the Implicit Quadrature is even better.

The Adams-Bashforth-Moulton is a combined method made up of the Adams-Bashforth method and Adams-Moulton method. Additionally, the Adams-Bashforth becomes the predictor whiles Adams-Moulton becomes the corrector in a multistep process to approximate the solution of the differential equation. With the advantage of adaptability and nonlocality exhibited by fractional order derivatives, the ABM method is able to simulate better and flexible than the traditional derivatives. Hence, in this paper, the Adams-Bashforth-Moulton predictor corrector numerical scheme is chosen for the numerical design and the approximate solution of the Caputo fractional derivative unemployment model developed by (Nti et al. 2024).

3 Development of the Numerical Scheme

The Caputo unemployment model developed by Nti et al (2024) is:

$$\left.\begin{aligned} {}^{C}D_t^{\alpha}\frac{dU}{dt} &= \Lambda + \varepsilon E + \rho(1-r)A - \psi\omega UA - (\eta+\tau+\sigma+\kappa)U \\ {}^{C}D_t^{\alpha}\frac{dE}{dt} &= \eta U - (\varepsilon+\kappa)E \\ {}^{C}D_t^{\alpha}\frac{dA}{dt} &= \psi\omega UA - \rho(1-r)A - (\theta\phi+\kappa)A \\ {}^{C}D_t^{\alpha}\frac{dV}{dt} &= \theta\phi A + (\tau+\sigma)U - \kappa V \end{aligned}\right\} \tag{1}$$

Where α is a fractional order with $0 < \alpha \leq 1$. Also, the initial values or conditions associated to the model variables in the fractional order model in equation (1) is given by:

$$U_0 = U(0) \geq 0, E_0 = E(0) \geq 0, A_0 = A(0) \geq 0, V_0 = V(0) \geq 0$$

The definition of the Caputo derivative at alpha greater than zero is:

$${}_{0}^{C}D_{\psi}^{\alpha} = \frac{1}{\Gamma(\alpha)}\int_0^{\psi}(\psi - s)^{\alpha-1} f(s, m(s)ds \tag{2}$$

The problem in equation (1) is primarily summarised in the procedure of a fractional order of differential systems of equations as:

$$D_{\psi}^{\alpha} m(\psi) = f(m, m(\psi)), \forall \psi \in [0, \psi] \tag{3}$$

And,

$m^{(q)}(0) = m_0^q$ where $q = 0,1,..., p-1$ and $p = [\alpha]$

Describing equation (3) in the form of Volterra integral gives:

$$m(\psi) = \sum_{q=0}^{p-1} m_0^q \frac{\psi^q}{q!} + \frac{1}{\Gamma(\alpha)} \int_0^{\psi} (\psi - s)^{\alpha-1} f(s, m(s)ds \tag{4}$$

The next thing is to put $h = \dfrac{\psi}{N}, \psi_n = nh, \forall n = 0,1,..., N \in ¢^+$

As seen from equation (4), the iterative formula is obtained from equation (4) as:

$$m(\psi_{n+1}) = \sum_{q=0}^{p-1} m_0^q \frac{\psi_{n+1}^q}{q!} + \frac{1}{\Gamma(\alpha)} \left[\frac{h^{\alpha}}{\alpha(\alpha+1)} \sum_{k=0}^{n+1} g_{k,n+1} f(\psi_k, m(\psi_k) \right] \tag{5}$$

Equation (5) gives:

$$m(\psi_{n+1}) = \sum_{q=0}^{p-1} m_0^q \frac{\psi_{n+1}^q}{q!} + \frac{h^{\alpha}}{\Gamma(\alpha+2)} \sum_{k=0}^{n} g_{k,n+1} f\left(\psi_k, m(\psi_k)\right) + \frac{h^{\alpha}}{\Gamma(\alpha+2)} f\left(\psi_{n+1}, m^a(\psi_{n+1})\right) \tag{6}$$

From equation (6), $m^a(\psi_{n+1})$ becomes the predicted value, and is given by:

$$m^a(\psi_{n+1}) = \sum_{q=0}^{p-1} m_0^q \frac{\psi_{n+1}^q}{q!} + \frac{1}{\Gamma(\alpha)} \sum_{k=0}^{n} d_{k,n+1} f\left(\psi_k, m(\psi_k)\right) \tag{7}$$

where,

$$g_{k,n+1} = \begin{cases} n^{(\alpha+1)} - (n+1)^{\alpha}(n-\alpha) & if \quad k = 0 \\ (n-k)^{(\alpha+1)} + (n-k+2)^{\alpha+1} - 2(n-k+1)^{\alpha+1} & if \quad 1 \le k \le n \\ 1 & if \quad k = n+1 \end{cases} \tag{8}$$

and,

$$d_{k,n+1} = (n-k+1)^{\alpha} - (n-k)^{\alpha} \quad if \quad 0 \le k \le n \tag{9}$$

The application of the above stated numerical scheme to the Caputo unemployment model (1), gives the system's discretized form as:

$$U(\psi_{n+1}) = \left.\begin{aligned} & U(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\Lambda + \varepsilon E^a + \rho(1-r)A^a - \psi\omega U^a A^a - (\eta+\tau+\sigma+\kappa)U^a\right] \\ & + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{k=0}^{m} g_{k,n+1}\left[\Lambda + \varepsilon E + \rho(1-r)A - \psi\omega UA - (\eta+\tau+\sigma+\kappa)U\right] \end{aligned}\right\} \tag{10}$$

$$E(\psi_{n+1}) = \left.\begin{aligned} & E(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\eta U^a - (\varepsilon+\kappa)E^a\right] \\ & + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{k=0}^{m} g_{k,n+1}\left[\eta U - (\varepsilon+\kappa)E\right] \end{aligned}\right\} \tag{11}$$

$$A(\psi_{n+1}) = \left.\begin{aligned} & A(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\psi\omega U^a A^a - \rho(1-r)A^a - (\theta\phi+\kappa)A^a\right] \\ & + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{k=0}^{m} g_{k,n+1}\left[\psi\omega UA - \rho(1-r)A - (\theta\phi+\kappa)A\right] \end{aligned}\right\} \tag{12}$$

$$V(\psi_{n+1}) = \left.\begin{aligned} & V(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\theta\phi A^a + (\tau+\sigma)U^a - \kappa V^a\right] \\ & + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{k=0}^{m} g_{k,n+1}\left[\theta\phi A + (\tau+\sigma)U - \kappa V\right] \end{aligned}\right\} \tag{13}$$

where,

$$U^a(\psi_{n+1}) = U(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+1)}\sum_{k=0}^{n} d_{k,n+1}\left(\Lambda + \varepsilon E + \rho(1-r)A - \psi\omega UA - (\eta+\tau+\sigma+\kappa)U\right) \tag{14}$$

$$E^a(\psi_{n+1}) = E(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+1)}\sum_{k=0}^{n} d_{k,n+1}\left(\eta U - (\varepsilon+\kappa)E\right) \tag{15}$$

$$A^a(\psi_{n+1}) = A(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+1)}\sum_{k=0}^{n} d_{k,n+1}\left(\psi\omega UA - \rho(1-r)A - (\theta\phi+\kappa)A\right) \tag{16}$$

$$V^a(\psi_{n+1}) = V(\psi_0) + \frac{h^\alpha}{\Gamma(\alpha+1)}\sum_{k=0}^{n} d_{k,n+1}\left(\theta\phi A + (\tau+\sigma)U - \kappa V\right) \tag{17}$$

4 Conclusions

In this research work, Adams-Bashforth-Moulton predictor corrector method has been employed to develop a novel numerical scheme that can be used as an iterative formula in the discretized form for the developed systems of equation of the Caputo unemployment model for simulations purposes to help understand the dynamics of unemployment in Ghana’s labor market.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generators have been used during the writing or editing of this manuscript.

Competing Interests

Authors have declared that no competing interests exist.

References

Al-Maalwi, R., Al-Sheikh, S., Ashi, H. A., & Asiri, S. (2021). Mathematical modeling and parameter estimation of unemployment with the impact of training programs. *Mathematics and Computers in Simulation, 182*, 705-720.
Available:https://doi.org/10.1016/j.matcom.2020.12.016

Aminikhah, H., Sheikhani, A. H. R., & Rezazadeh, H. (2018). Approximate analytical solutions of distributed order fractional Riccati differential equation. *Ain Shams Engineering Journal, 9*(4), 581-588.
Available:https://doi.org/10.1016/j.asej.2017.06.002

Ashi, H. A., Al-Maalwi, R. M., & Al-Sheikh, S. (2022). Study of the unemployment problem by mathematical modeling: Predictions and controls. *The Journal of Mathematical Sociology, 46*(3), 301-313.
Available:https://doi.org/10.1080/0022250X.2022.2064603

Ayoade, A., Odetunde, O., & Falodun, B. (2020). Modeling and analysis of the impact of vocational education on the unemployment rate in Nigeria. *Applications and Applied Mathematics: An International Journal (AAM), 15*(1), 550-564.
Available: https://digitalcommons.pvamu.edu/aam/vol15/iss1/32

Blank, L. (1997). Numerical treatment of differential equations of fractional order. *Non-linear World, 4*(4), 473-491.

Diethelm, K. (1997). An algorithm for the numerical solution of differential equations of fractional order. *Electronic Transactions on Numerical Analysis, 5*(1), 1-6.

Diethelm, K., & Ford, N. (2002). A predictor-corrector approach for the numerical solution of fractional differential equations. *Nonlinear Dynamics, 29*, 3–22.
Available:https://doi.org/10.1023/A:1016592219341

Diethelm, K., & Luchko, Y. (2004). Numerical solution of linear multi-term initial value problems of fractional order. *Journal of Computational and Applied Analysis, 6*(3), 243-263.
Available:https://doi.org/10.3934/jcaa.2004.6.243

Diethelm, K., Ford, N. J., & Freed, A. D. (2002). A predictor-corrector approach for the numerical solution of fractional differential equations. *Nonlinear Dynamics, 29*, 3-22.
Available:https://doi.org/10.1023/A:1016592219341

Diethelm, K., Ford, N. J., & Freed, A. D. (2004). Detailed error analysis for a fractional Adams method. *Numerical Algorithms, 36*, 31-52.
Available:https://doi.org/10.1023/B:NUMA.0000049290.63573.72

Ford, N. J., & Connolly, J. A. (2006). Comparison of numerical methods for fractional differential equations. *Communications on Pure and Applied Analysis, 5*(2), 289.
Available:https://doi.org/10.3934/cpaa.2006.5.289

Gorenflo, R. (1997). Fractional calculus: Some numerical methods (pp. 277-290). In *Fractional Calculus* (Vol. 5). Springer Vienna.
Available:https://doi.org/10.1007/978-3-7091-7032-3_16

Malinowska, A., & Torres, D. (2012). Towards a combined fractional mechanics and quantization. *Fractional Calculus and Applied Analysis*, *15*(3), 407-417.
Available:https://doi.org/10.2478/s13540-012-0024-5

Mallick, U. K., & Biswas, M. H. A. (2017). Optimal control strategies applied to reduce the unemployed population. In *Proceedings of the 5th IEEE R10-HTC, BUET, Dhaka* (pp. 408-411).
Available:https://doi.org/10.1109/R10-HTC.2017.8282202

Misra, A. K., & Singh, A. K. (2011). A mathematical model for unemployment. *Nonlinear Analysis: Real World Applications, 12*(1), 128-136.
Available:https://doi.org/10.1016/j.nonrwa.2010.05.001

Misra, A. K., & Singh, A. K. (2013). A delay mathematical model for the control of unemployment. *Differential Equations and Dynamical Systems, 21*(3), 291–307.
Available:https://doi.org/10.1007/s12591-013-0171-0

Momenzadeh, A., & Ahrabi, S. S. (2017). A comparison between numerical solutions to fractional differential equations: Adams-type predictor-corrector and multi-step generalized differential transform method. *arXiv preprint arXiv:1710.04183*.
Available:https://arxiv.org/abs/1710.04183

Munoli, S. B., & Gani, S. R. (2016). Optimal control analysis of a mathematical model for unemployment. *Optimal Control Applications and Methods, 37*(4), 798-806.
Available:https://doi.org/10.1002/oca.2247

Munoli, S. B., Gani, S., & Gani, S. R. (2017). A mathematical approach to employment policies: An optimal control analysis. *International Journal of Statistics and Systems, 12*(3), 549-565.
Available:https://doi.org/10.2991/ijss.12.3.8

Nikolopoulos, C. V., & Tzanetis, D. E. (2003). A model for housing allocation of a homeless population due to a natural disaster. *Nonlinear Analysis: Real World Applications, 4*, 561–579.
Available:https://doi.org/10.1016/S1468-1218(02)00119-5

Nti, A. E., Acquah, J., & Otoo, H. (2024). Compartmental model for unemployment dynamics in Ghana using Caputo fractional derivative. *Asian Research Journal of Mathematics, 20*(10), 86-100.
Available:https://doi.org/10.9734/arjom/2024/v20i10848

Omame, A., Abbas, M., & Onyenegecha, C. P. (2022). A fractional order model for the co-interaction of COVID-19 and Hepatitis B virus. *Results in Physics, 37*, 105498.
Available:https://doi.org/10.1016/j.rinp.2022.105498

Pathan, G., & Bathawala, P. H. (2015). A mathematical model for unemployment problem with effect of self-employment. *IOSR Journal of Mathematics (ISOR-JM), 11*(6), 37-43.
Available:https://doi.org/10.9790/5728-11623743

Roman, P. (2021). Application of the Adams-Bashforth-Moulton method to the numerical study of linear fractional oscillators models. In *AIP Conference Proceedings* (Vol. 2365, No. 1). AIP Publishing.
Available:https://doi.org/10.1063/5.0050544

Rosa, S., & Torres, D. F. (2023). Numerical fractional optimal control of respiratory syncytial virus infection in Octave/Matlab. *Mathematics, 11*(6), 1511.
Available:https://doi.org/10.3390/math11061511

Rosenfeld, J. A., & Dixon, W. E. (2017). Approximating the Caputo fractional derivative through the Mittag-Leffler reproducing kernel Hilbert space and the kernelized Adams–Bashforth–Moulton method. *SIAM Journal on Numerical Analysis*, *55*(3), 1201-1217. Available:https://doi.org/10.1137/16M1075689

Shabani, A., Sheikhani, A. R., Aminikhah, H., & Gholamin, P. (2021). A predictor-corrector scheme for the nonlinear chaotic variable-order fractional three-dimensional system. *University Politehnica of Bucharest Scientific Bulletin-Series A-Applied Mathematics and Physics, 83*(2), 187-202.

Toufik, M., & Atangana, A. (2017). New numerical approximation of fractional derivative with non-local and non-singular kernel: Application to chaotic models. *The European Physical Journal Plus*, *132*, 1-16. Available:https://doi.org/10.1140/epjp/i2017-11521-7

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